Complex Numbers

In mathematics, complex numbers are used to represent numbers with negative roots. We make use of the constant $i = \sqrt{-1}$ to represent complex numbers[1]. The following notations are used for complex numbers:

- **Cartesian**: $a + ib$
- **Polar**[2]: $re^{i\theta} = r(\cos\theta + i\sin\theta) = r\cos\theta + ir\sin\theta$

You can convert between Cartesian and Polar form by making use of the relations:

$$a = r\cos\theta$$

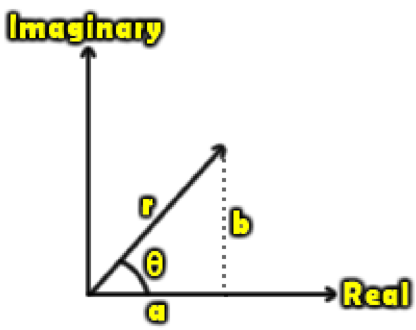
$$b = r\sin\theta$$

$$r = \sqrt{a^2 + b^2}$$

From the above relation, a is referred to as the real part of a complex number, and b is referred to as the imaginary part of the complex number. In electronics, **Phasor Notation** is commonly used. It is defined as follows:

$$r\angle\theta = re^{i\theta} = r(\cos\theta + i\sin\theta)$$

Complex numbers can be graphically represented as follows:



It is easy to see from the graph that a vector of magnitude r is formed from the real (a) and imaginary (b) components of the complex number, with an angle θ between these components. The magnitude of r is then simply derived from the Pythagorean Theorem due to the fact that this is a right-angled triangle: $r = \sqrt{a^2 + b^2}$. The angle θ between the components can also be derived from trigonometry:

$$\tan\theta = \frac{b}{a} = \frac{r\sin\theta}{r\cos\theta} = \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

It is important to be aware that a complex number represents a vector. Furthermore, the angle θ always starts from the real axis and moves counter-clockwise until it reaches r. Complex numbers can be added, subtracted, multiplied, and divided just like any other number.

[1]Note that $i^2 = \sqrt{-1}\sqrt{-1} = \left(\sqrt{-1}\right)^2 = \left((-1)^{\frac{1}{2}}\right)^2 = -1$.

[2]Note that $e^{i\theta} = \cos\theta + i\sin\theta$ is Euler's Formula. I have shown a proof for it in another tutorial.